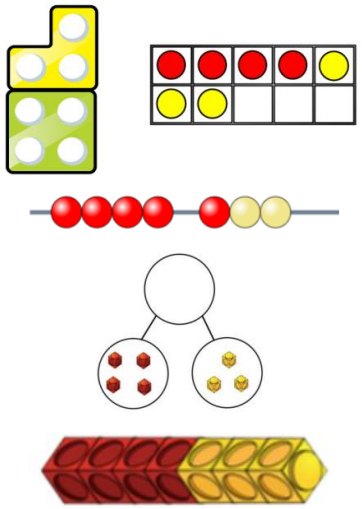
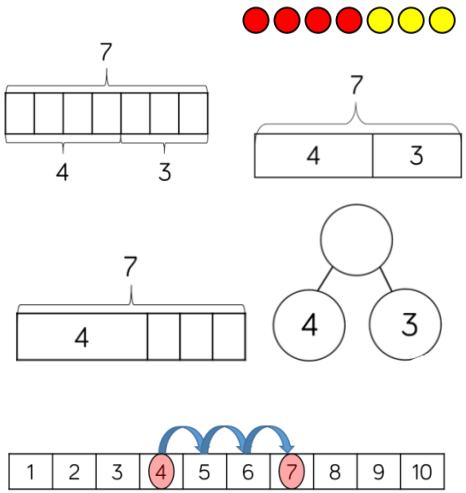
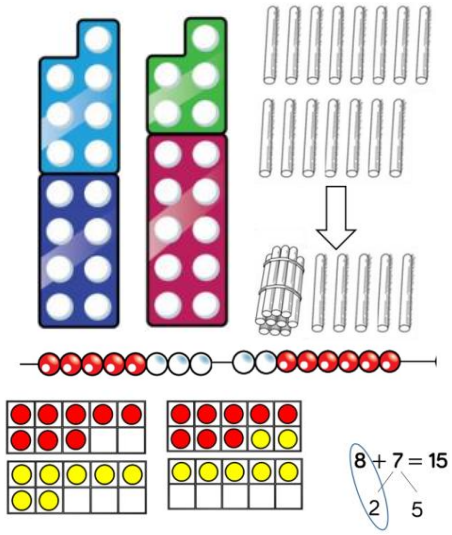
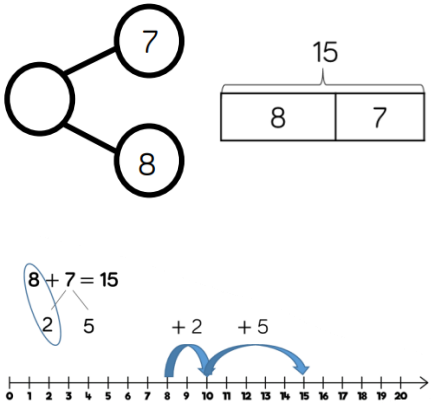
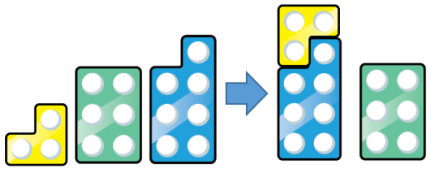
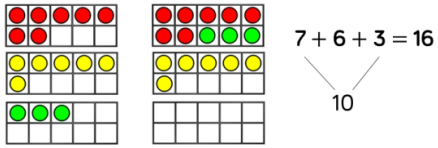
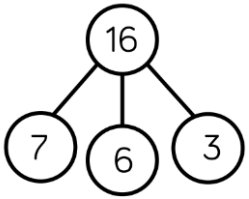
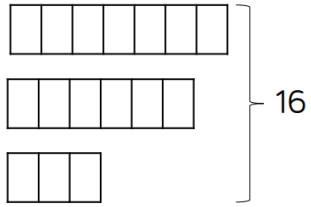
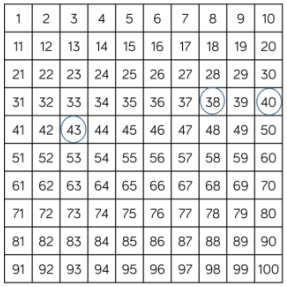
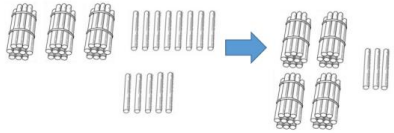
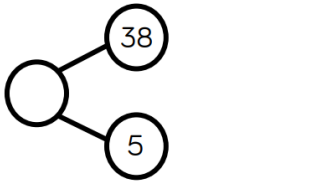
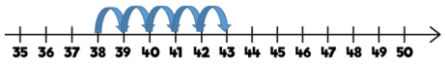
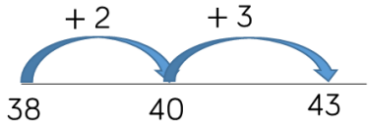
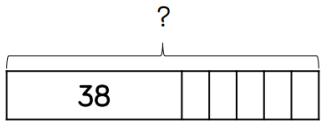
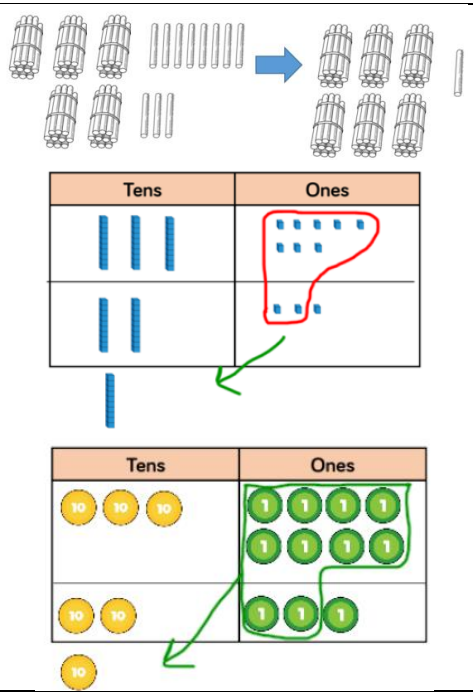
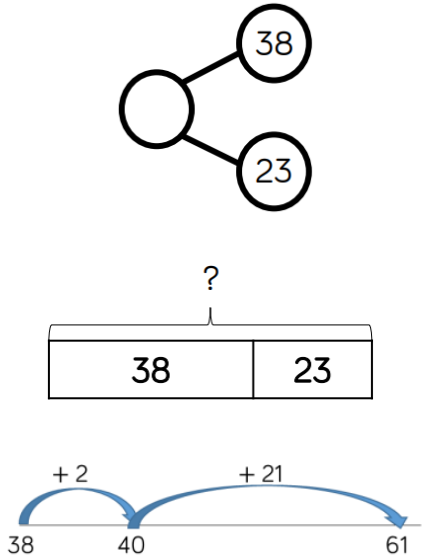
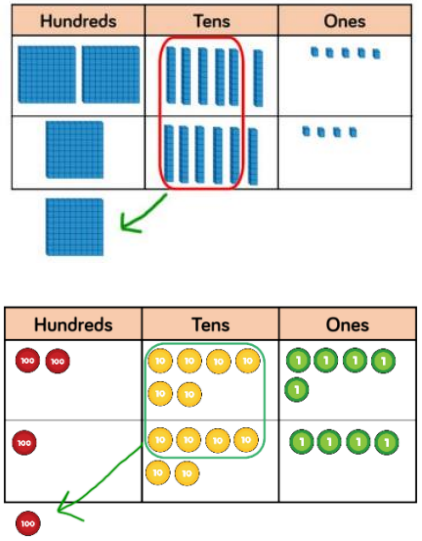
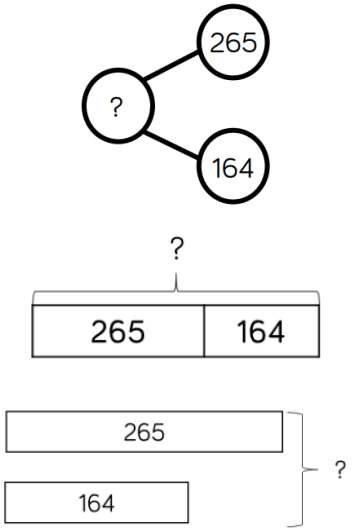
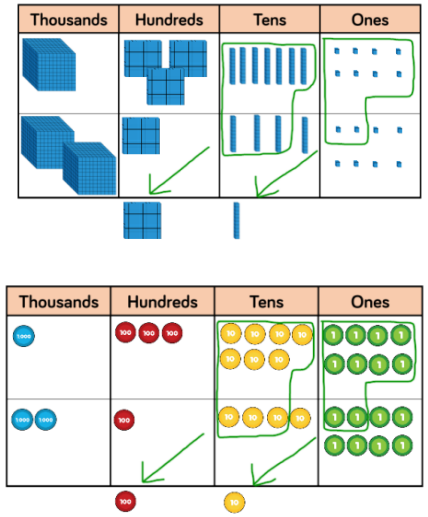
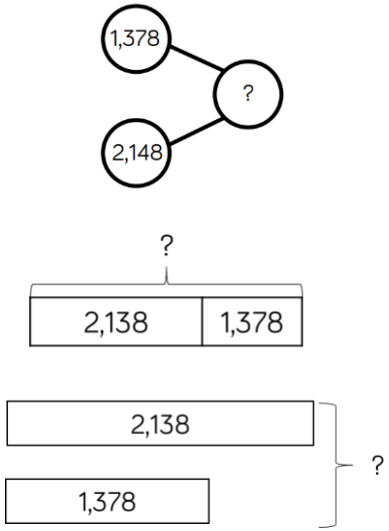
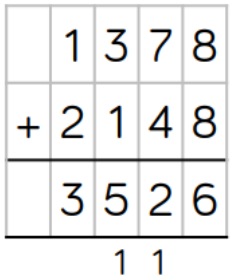
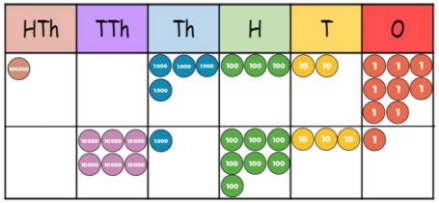
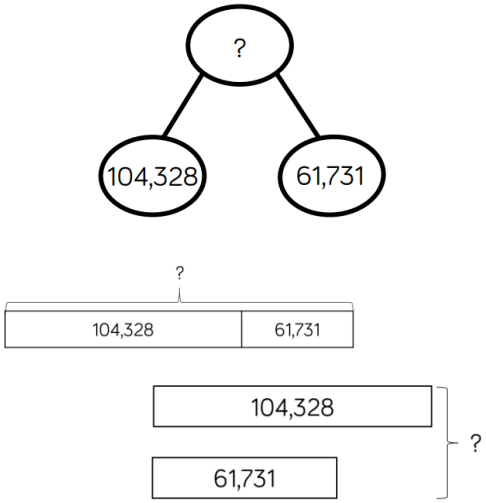
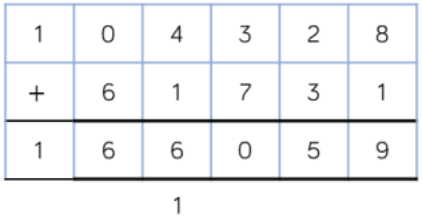
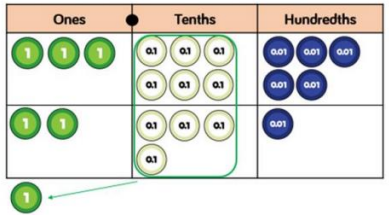
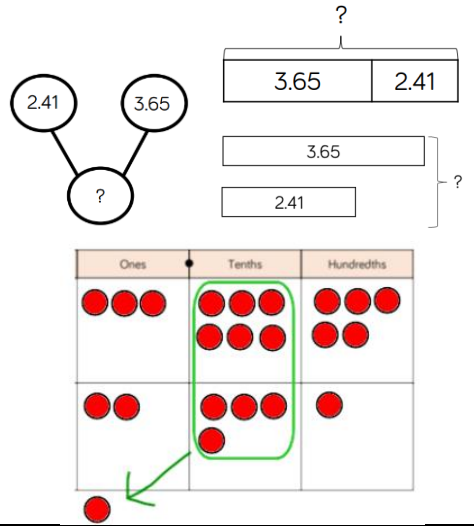
ADDITION

Skill	Notes	Concrete	Pictorial	Abstract
Add 1-digit numbers within 10 (Year 1)	When adding numbers to 10, children can explore both aggregation and augmentation. The part-whole model, discrete and continuous bar model, number shapes and ten frame support aggregation. The combination bar model, ten frame, bead string and number track all support augmentation.			
Add 1 and 2 -digit numbers to 20 (Year 1/2)	When adding one - digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten. In Year 1, this is only done just by counting on. From Year 2, use different manipulatives can be used to represent this exchange alongside number lines to support children in understanding how to partition their jumps.			

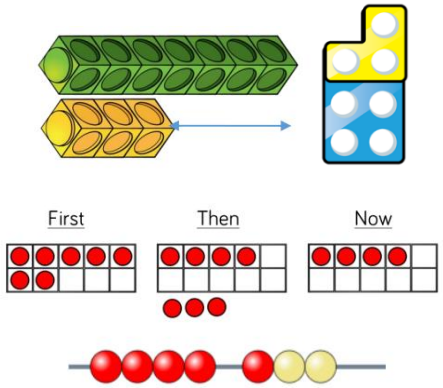
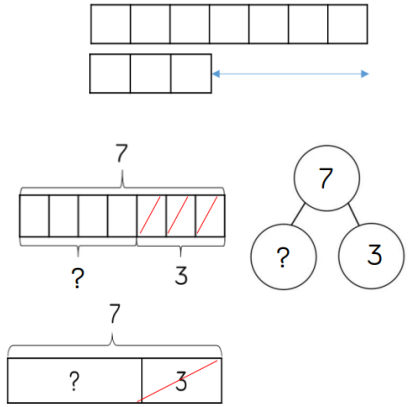
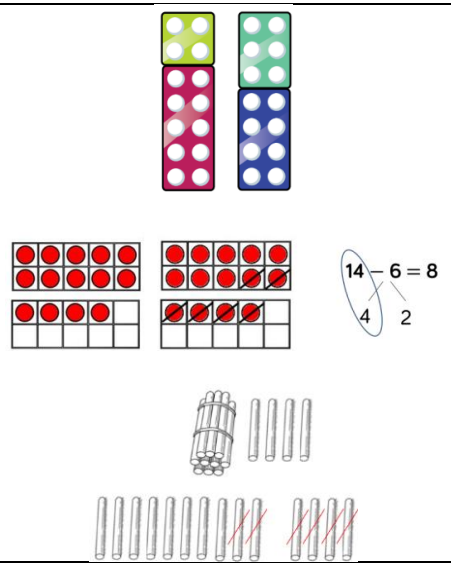
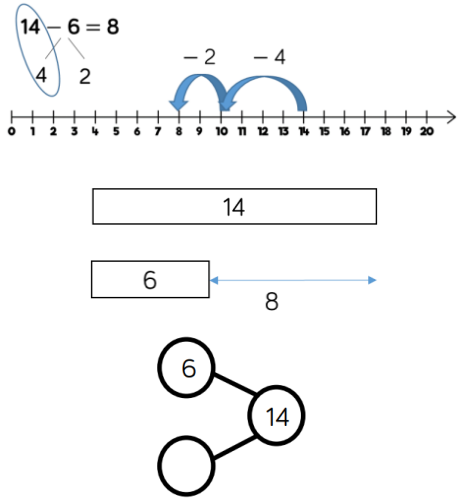
Add three 1 -digit numbers (Year 2)	When adding three 1 - digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently. This supports children in their understanding of commutativity. Manipulatives that highlight number bonds to 10 are effective when adding three 1 - digit numbers.	 	 	
Add 1-digit and 2-digit numbers to 100 (Year 2/3)	When adding single digits to a two-digit number, children should be encouraged to count on from the larger number. They should also apply their knowledge of number bonds to add more efficiently e.g. 8 + 5 = 13 so 38 + 5 = 43. Hundred squares and straws can support children to find the number bond to 10.	 	   	

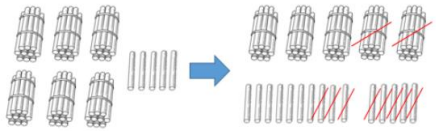
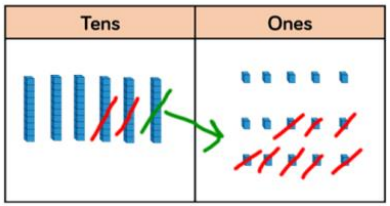
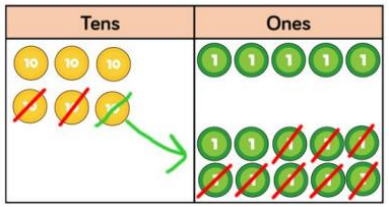
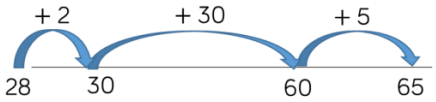
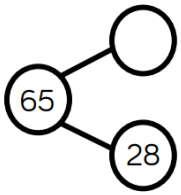
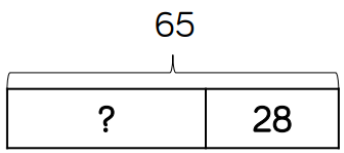
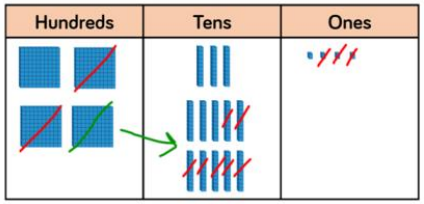
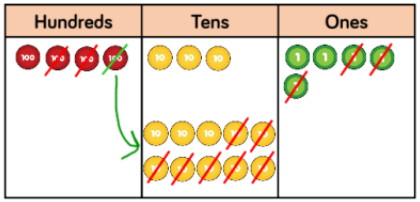
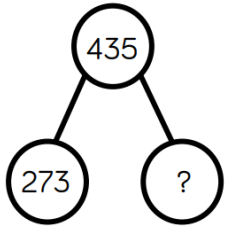
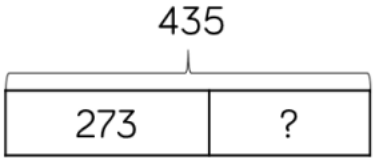
Add two 2-digit numbers to 100 (Year 2/3)	Children can use a blank number line and other representations to count on to find the total. Encourage them to jump to multiples of 10 to become more efficient. From Year 3, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.			$$\begin{array}{r} 38 \\ +\ 23 \\ \hline 61 \\ \hline 1 \end{array}$$
Add numbers with up to 3 digits (Year 3)	Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 3 digits. Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method. Plain counters on a place value grid can also be used to support learning.			$$\begin{array}{r} 265 \\ +\ 164 \\ \hline 429 \\ \hline 1 \end{array}$$

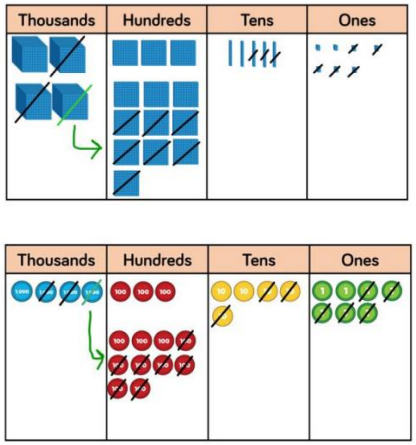
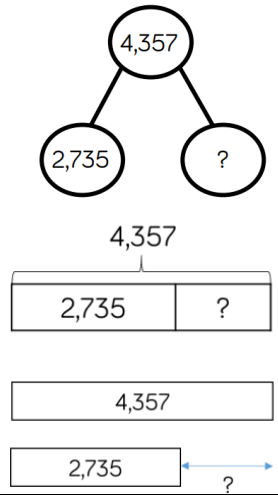
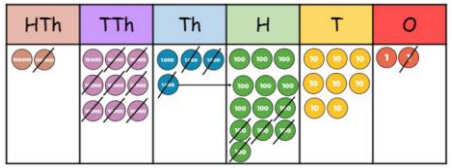
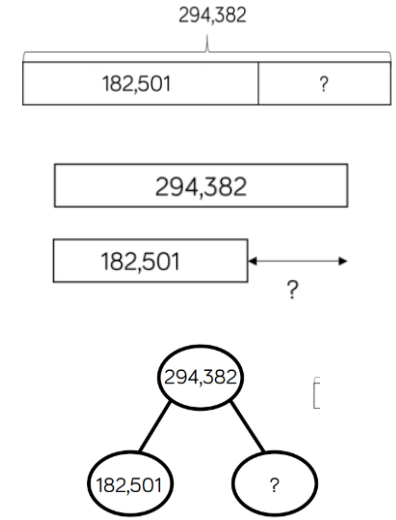
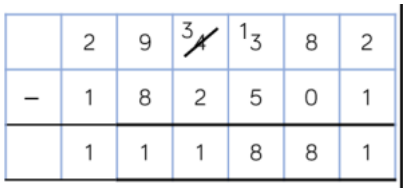
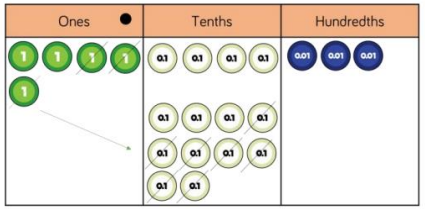
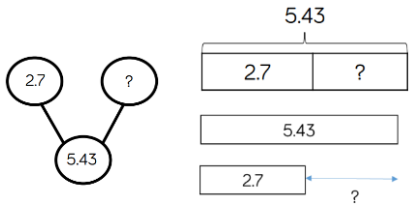
Add numbers with up to 4 digits (Year 4)	Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 4 digits. Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method. Plain counters on a place value grid can also be used to support learning.			
Add numbers with more than 4 digits (Year 5/6)	Place value counters or plain counters on a place value grid are the most effective concrete resources when adding numbers with more than 4 digits. At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.			

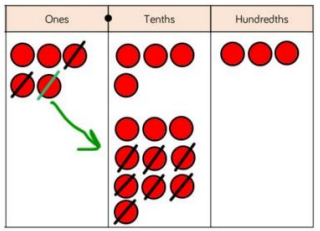
Add with up to 3 decimal places (Year 5)	Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places. Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.			$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$

SUBTRACTION

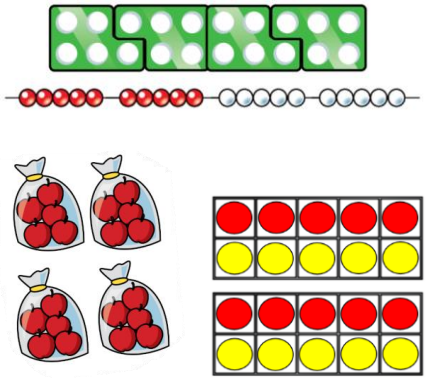
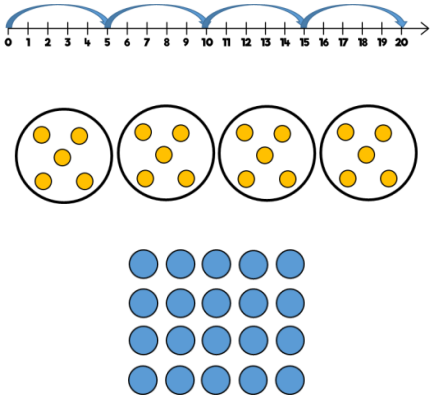
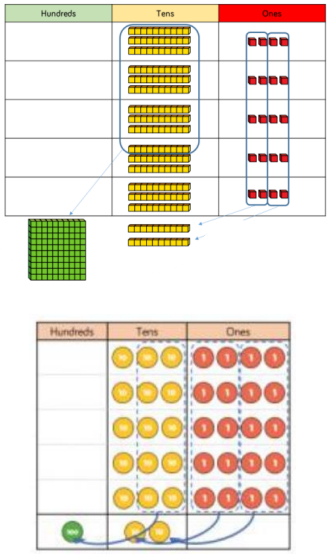
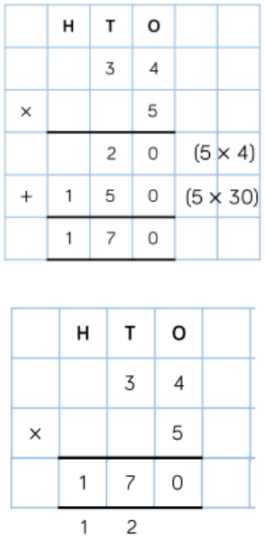
Skill	Notes	Concrete	Pictorial	Abstract
Subtract 1-digit numbers within ten (Year 1)	Part-whole models, bar models, ten frames and number shapes support partitioning. Ten frames, number tracks, single bar models and bead strings support reduction. Cubes and bar models with two bars can support finding the difference.			
Subtract 1 and 2-digit numbers to 20. (Year 1/2)	In Year 1, subtracting one-digit numbers that cross 10, is done by counting back, using objects, number tracks and number lines. From Year 2, children should be encouraged to find the number bond to 10 when partitioning the subtracted number.			

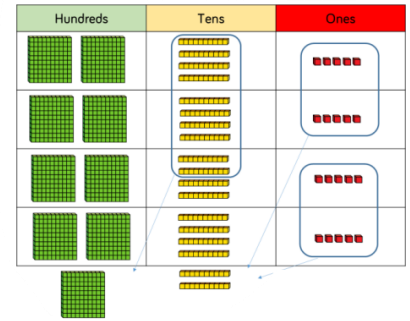
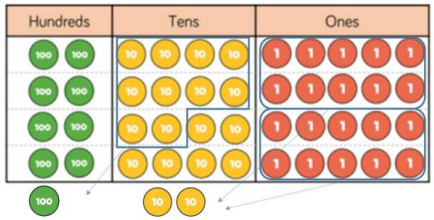
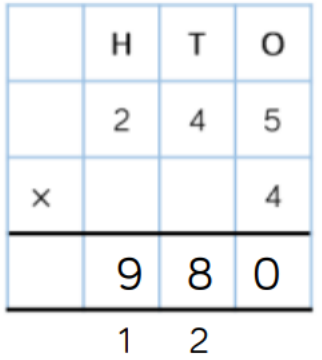
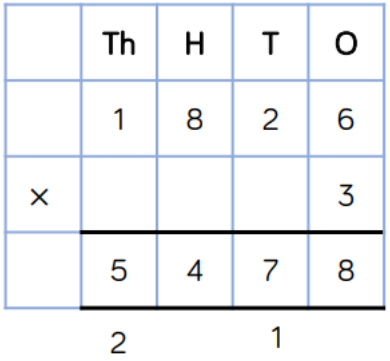
Subtract 1 and 2-digit numbers to 100. **(Year 2/3)**	Children can also use a blank number line to count back to find the difference. Encourage them to jump to multiples of 10 to become more efficient. From Year 3, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.	   	   	5 1 65 − 28 37
Subtract numbers with up to 3 digits **(Year 3)**	Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 3 digits. Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method. Plain counters on a place value grid can also be used to support learning.	  	  	3 1 435 − 273 162

Subtract numbers with up to 4 digits (Year 4)	Base 10 and place value counters are the most effective manipulatives when subtracting numbers with up to 4 digits. Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method. Plain counters on a place value grid can also be used to support learning			3 1 4357 − 2735 1622
Subtract numbers with more than 4 digits (Year 5/6)	Place value counters or plain counters on a place value grid are the most effective concrete resource when subtracting numbers with more than 4 digits. At this stage, children should be encouraged to work in the abstract, using column method to subtract larger numbers efficiently.			
Subtract with up to 3 decimal places (Year 5/6)	Place value counters and plain counters on a place value grid are the most effective manipulative when subtracting decimals with 1, 2 and then 3 decimal places. Ensure children have experience of			4 1 5.43 − 2.7 2.73

	subtracting decimals with a variety of decimal places. This includes putting this into context when subtracting money and other measures.			

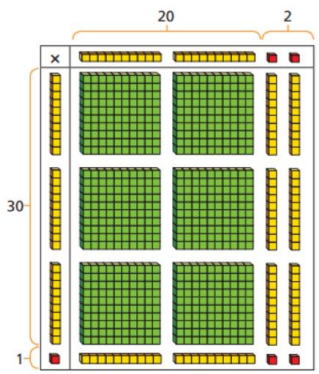
MULTIPLICATION

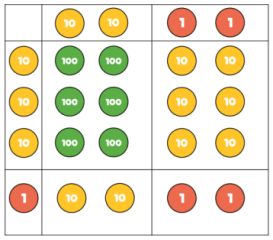
Skill	Notes	Concrete	Pictorial	Abstract
Solve 1-step problems using multiplication (Year 1/2) Eg. One bag holds 5 apples. How many apples do 4 bags hold?	Children represent multiplication as repeated addition in many different ways. In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record multiplication formally. In Year 2, children are introduced to the multiplication symbol.			$5 + 5 + 5 + 5 = 20$ $4 \times 5 = 20$ $5 \times 4 = 20$
Multiply 2-digit numbers by 1-digit numbers (Year 3/4)	Informal methods and the expanded method are used in Year 3 before moving on to the short multiplication method in Year 4. Place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge.		Pictoral representations of concrete methods.	

Multiply 3-digit numbers by 1-digit numbers (Year 4)	When moving to 3- digit by 1-digit multiplication, encourage children to move towards the short, formal written method. Base 10 and place value counters continue to support the understanding of the written method. Limit the number of exchanges needed in the questions and move children away from resources when multiplying larger numbers.	 	Pictoral representations of concrete methods.	
Multiply 4-digit numbers by 1-digit numbers (Year 5)	When multiplying 4-digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method. If children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so children can focus on the use of the written method.		Pictoral representations of concrete methods.	

<table>
<tr>
<td>Multiply 2-digit numbers by 2-digit numbers
(Year 5)</td>
<td>When multiplying a multi-digit number by 2-digits, use the area model to help children understand the size of the numbers they are using.

This links to finding the area of a rectangle by finding the space covered by the Base 10.

The grid method matches the area model as an initial written method before moving on to the formal written multiplication method.</td>
<td> 

 </td>
<td>Pictoral representations of concrete methods.</td>
<td>
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<tr><td>×</td><td>20</td><td>2</td></tr>
<tr><td>30</td><td>600</td><td>60</td></tr>
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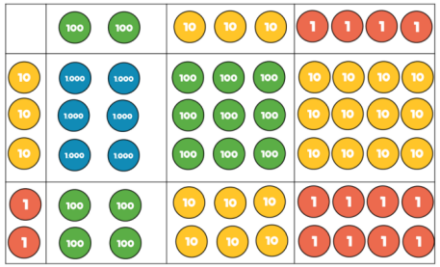
<tr><td></td><td></td><td>2</td><td>2</td></tr>
<tr><td></td><td>6</td><td>6</td><td>0</td></tr>
<tr><td></td><td>6</td><td>8</td><td>2</td></tr>
</table>
</td>
</tr>
<tr>
<td>Multiply 3-digit numbers by 2-digit numbers
(Year 5)</td>
<td>Children can continue to use the area model when multiplying 3-digits by 2-digits.

Place value counters become more efficient to use but Base 10 can be used to highlight the size of numbers.

Children should now move towards the formal written method, seeing the links with the grid method.</td>
<td>  </td>
<td>Pictoral representations of concrete methods.</td>
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<tr><td>30</td><td>6,000</td><td>900</td><td>120</td></tr>
<tr><td>2</td><td>400</td><td>60</td><td>8</td></tr>
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<tr><td>×</td><td></td><td>3</td><td>2</td></tr>
<tr><td></td><td>4</td><td>6</td><td>8</td></tr>
<tr><td>1 7</td><td>1 0</td><td>2</td><td>0</td></tr>
<tr><td>7</td><td>4</td><td>8</td><td>8</td></tr>
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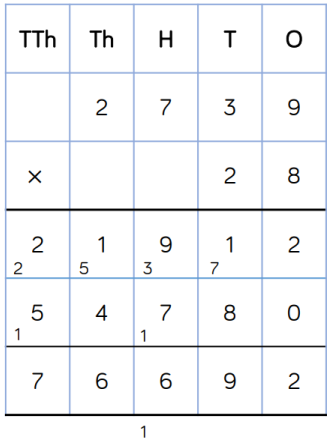
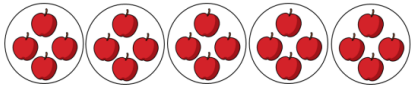
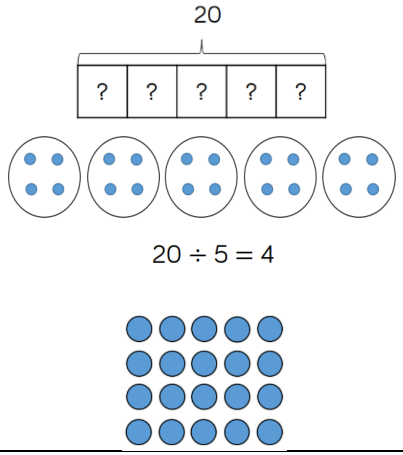
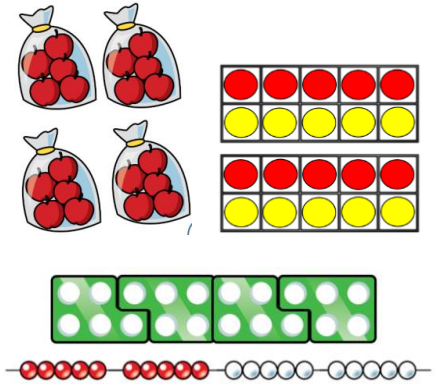
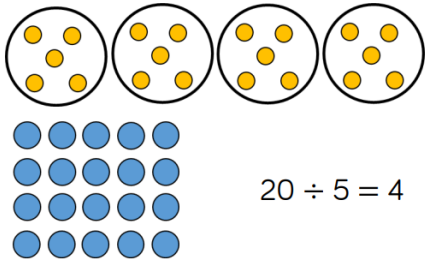
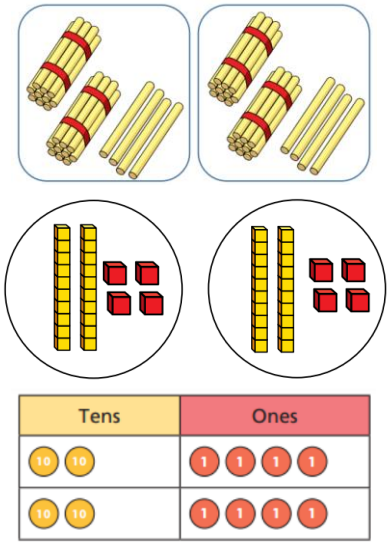
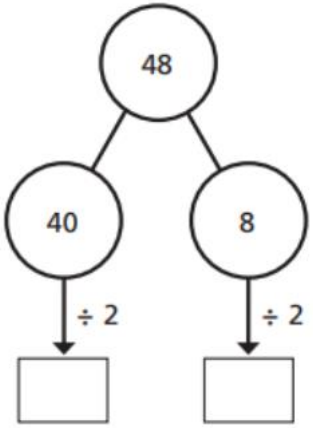
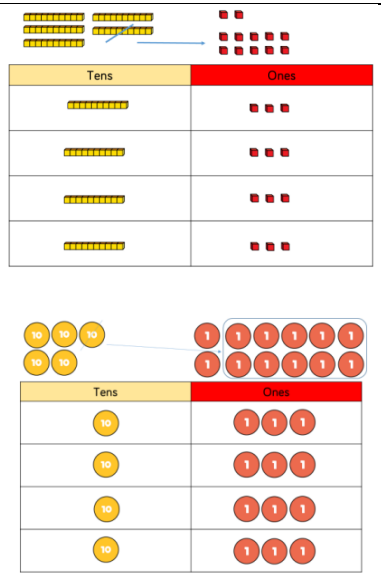
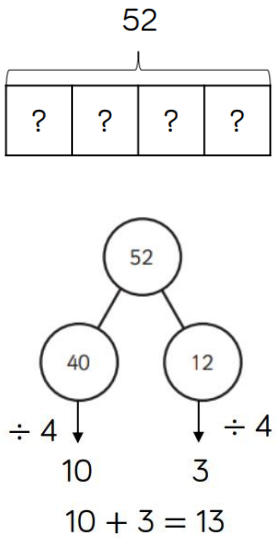
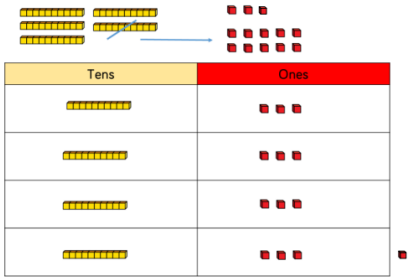
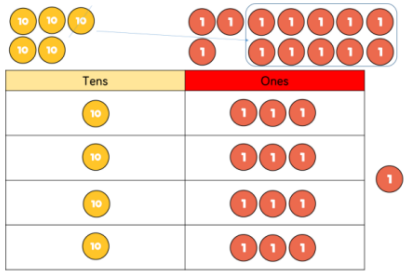
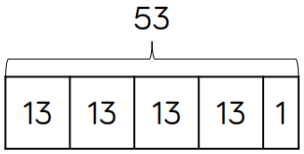
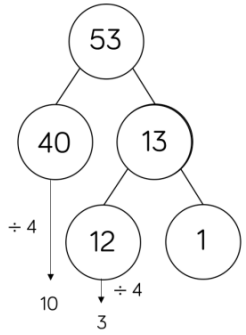
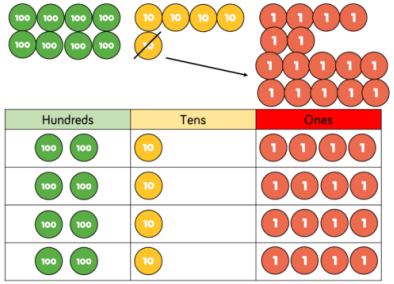
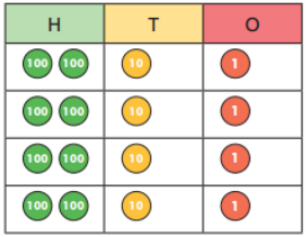
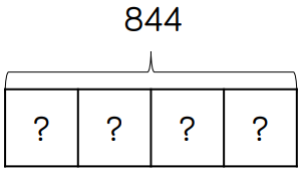
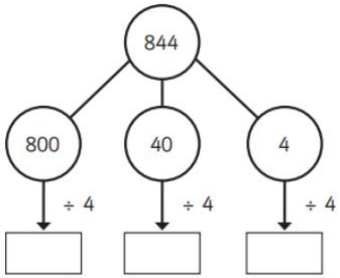
</table>

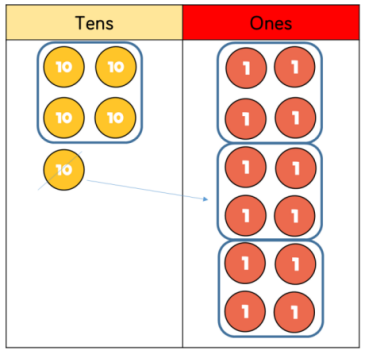
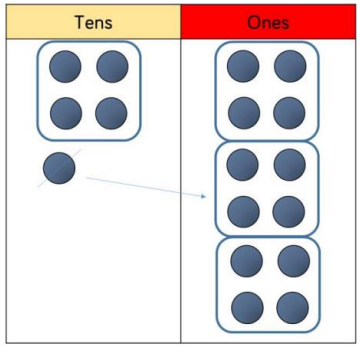
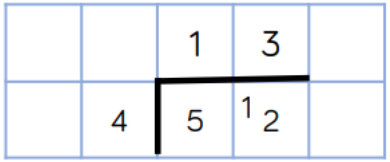
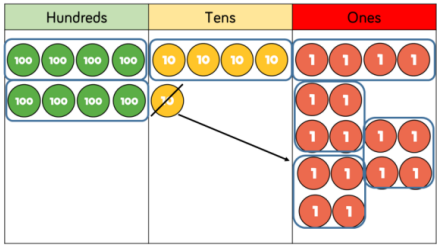
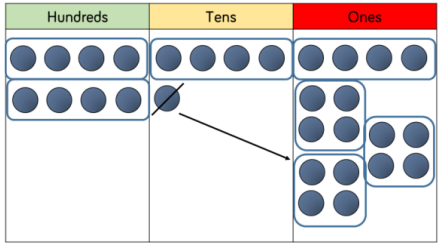
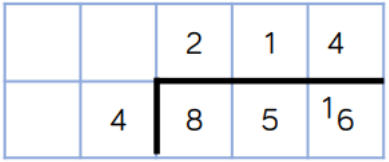
Multiply 4-digit numbers by 2-digit numbers (Year 5/6)	When multiplying 4-digits by 2-digits, children should be confident in using the formal written method. If they are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method. Consider where exchanged digits are placed and make sure this is consistent.			

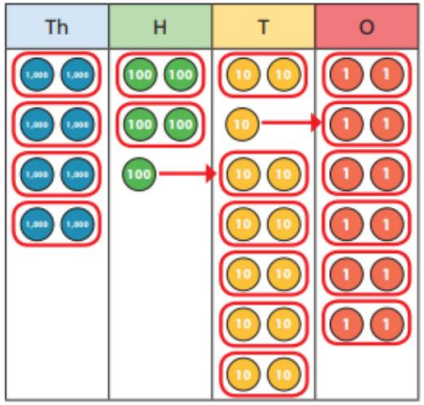
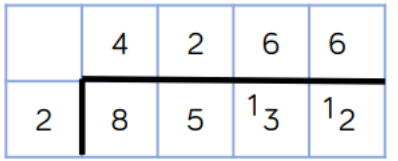
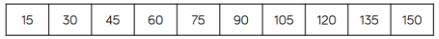
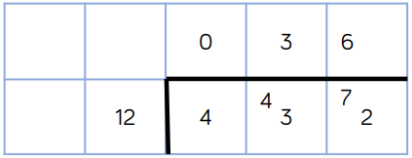
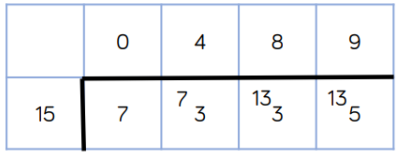
DIVISION

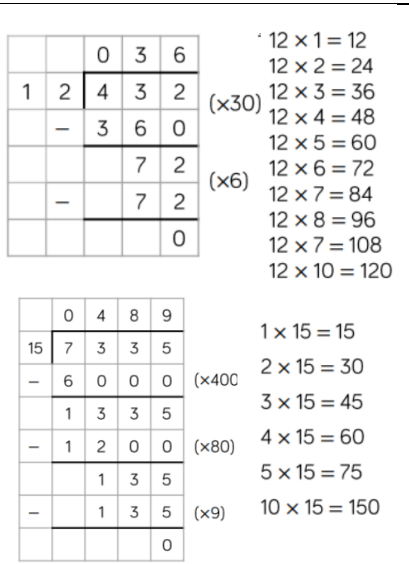
Skill	Notes	Concrete	Pictorial	Abstract
Solve 1-step problems using multiplication (sharing) (Year 1/2) Eg. There are 20 apples altogether. They are shared equally between 5 bags. How many apples are in each bag?	Children solve problems by sharing amounts into equal groups. In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record division formally. In Year 2, children are introduced to the division symbol.			
Solve 1-step problems using division (grouping) (Year 1/2) Eg. There are 20 apples altogether. They are put in bags of 5. How many bags are there?	Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as number shapes which helps to show the link between multiplication and division.			

Divide 2-digits by 1-digit (sharing with no exchange) (Year 3)	When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones. Straws, Base 10 and place value counters can all be used to share numbers into equal groups. Part-whole models can provide children with a clear written method that matches the concrete representation.			
Divide 2-digits by 1-digit (sharing with exchange) (Year 3/4)	When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones. Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows. Flexible partitioning in a part-whole model supports this method.			

Divide 2-digits by 1-digit (sharing with remainders) (Year 3/4)	When dividing numbers with remainders, children can use Base 10 and place value counters to exchange one ten for ten ones. Starting with the equipment outside the place value grid will highlight remainders, as they will be left outside the grid once the equal groups have been made. Flexible partitioning in a part-whole model supports this method.	 	 	
Divide 3-digits by 1-digit (sharing) (Year 4)	Children can continue to use place value counters to share 3-digit numbers into equal groups. Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between the rows. This method can also help to highlight remainders. Flexible partitioning in a part-whole model supports this method.	 	 	

Divide 2-digits by 1-digit (grouping) (Year 5)	When using the short division method, children use grouping. Starting with the largest place value, they group by the divisor. Language is important here. Children should consider 'How many groups of 4 tens can we make?' and 'How many groups of 4 ones can we make?' Remainders can also be seen as they are left ungrouped.			
Divide 3-digits by 1-digit (grouping) (Year 5)	Children can continue to use grouping to support their understanding of short division when dividing a 3-digit number by a 1-digit number. Place value counters or plain counters can be used on a place value grid to support this understanding. Children can also draw their own counters and group them through a more pictorial method.			

Divide 4-digits by 1-digit (grouping) (Year 5)	Place value counters or plain counters can be used on a place value grid to support children to divide 4- digits by 1-digit. Children can also draw their own counters and group them through a more pictorial method. Children should be encouraged to move away from the concrete and pictorial when dividing numbers with multiple exchanges.		Pictoral representations of concrete methods.	
Divide multi digits by 2-digits (short division) (Year 6)	When children begin to divide up to 4- digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective. Children can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.			 

Divide multi-digits by 2-digits (long division) (Year 6)	Children can also divide by 2-digit numbers using long division. Children can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.			
Divide multi digits by 2-digits (long division) (Year 6)	When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction. This will depend on the context of the question. Children can also answer questions where the quotient needs to be rounded according to the context.